Linear Algebra and its Applications 432 (2010) 670–677

Contents lists available at ScienceDirect

Linear Algebra and its Applications

journal homepage: www.elsevier.com/locate/laa

Estimation of $\|A^{-1}\|_\infty$ for weakly chained diagonally dominant M-matrices ☆

Ting-Zhu Huang [a,*], Yan Zhu [b]

[a] School of Applied Mathematics, University of Electronic Science and Technology of China, Chengdu, Sichuan 610054, PR China
[b] College of Mathematic and Information Science, Qujing Normal University, Qujing, Yunnan 655011, PR China

A R T I C L E   I N F O

Article history:
Received 20 September 2008
Accepted 8 September 2009
Available online 9 October 2009

Submitted by L. Elsner

AMS classification:
15A42
15A45
65F15

Keywords:
Weakly chained diagonally dominant matrix
M-matrix
Norm

A B S T R A C T

Let A be a weakly chained diagonally dominant ($wcdd$) M-matrix, an upper bound for $\|A^{-1}\|_\infty$ is presented and further applied to establish an lower bound for the smallest eigenvalue of A. Effectiveness of the new upper bound is shown numerically.

1. Introduction

Let $R^{n\times n}$ be the set of all real matrices of order n, $A=(a_{ij})\in R^{n\times n}$, and N denote the set $\{1,2,\ldots,n\}$, for any positive integer n.

An $n\times n$ matrix A is called a nonsingular M-matrix if there exists an $n\times n$ nonnegative matrix B and some real number a such that

$$A=aI-B,\quad a>\rho(B),$$

☆ This research was supported by NSFC (60973015), the Chinese Universities Specialized Research Fund for the Doctoral Program (20070614001), Sichuan Province Sci. and Tech. Research Project (2009HH0025).

* Corresponding author.
E-mail addresses: tingzhuhuang@126.com (T.-Z. Huang), zhuyan-wuda@sina.com (Y. Zhu).

0024-3795/$ - see front matter
doi:10.1016/j.laa.2009.09.012

where I is the identity matrix, $\rho(B)$ is the spectral radius of the nonnegative matrix B. If $q(A)$ denotes the minimum of all real eigenvalues of an M-matrix A, then $q(A) = \frac{1}{\rho(A^{-1})}$, where $\rho(A^{-1})$ is the spectral radius of A^{-1}. For further discussion of this issue, see, for example, [1,2].

For convenience, some notations are introduced as follows

$$R_i(A) = \sum_{j=1,j\neq i}^{n} |a_{ij}|, \quad r_i(A) = \sum_{j=1}^{n} |a_{ij}|,$$

$$\rho_i = \frac{1}{|a_{ii}|}\sum_{j=1,j\neq i}^{n} |a_{ij}|, \quad l_i = \frac{1}{|a_{ii}|}\sum_{j=1}^{i-1} |a_{ij}|, \quad u_i = \frac{1}{|a_{ii}|}\sum_{j=i+1}^{n} |a_{ij}|,$$

$$b_k = \max\left\{\frac{\sum_{j\neq i+k,k\leqslant j\leqslant n} |a_{i+k,j}|}{|a_{i+k,i+k}|},\ i = 1,\ldots,n-k\right\}, \quad k = 1,\ldots,n,$$

$$p_k = \max\left\{\frac{|a_{i+k,k}| + \sum_{h=k+1,h\neq i+k}^{n} |a_{i+k,h}|b_k}{|a_{i+k,i+k}|},\ i = 1,\ldots,n-k\right\}, \quad k = 1,\ldots,n,$$

$$d_k = \max\left\{\frac{\sum_{j\neq i+k-1,k\leqslant j\leqslant n} |a_{i+k-1j}|}{|a_{i+k-1,i+k-1}|},\ i = 1,\ldots,n-k+1\right\}, \quad k = 1,\ldots,n.$$

Obviously, $\rho_i = (u_i + l_i)$.

In numerical analysis, a bound is often required for $\|A^{-1}\|_\infty$. It is well known that it is usually difficult to bound $\|A^{-1}\|$ in any norm unless A^{-1} is known explicitly.

For a strictly diagonally dominant matrix $A = (a_{ij}) \in R^{n\times n}$, i.e.,

$$|a_{ii}| > \sum_{j=1,j\neq i}^{n} |a_{ij}|, \quad i \in N,$$

Varga [3] obtained a bound of $\|A^{-1}\|_\infty$ as

$$\|A^{-1}\|_\infty \leqslant \max\left\{\frac{1}{|a_{ii}| - \sum_{j=1,j\neq i}^{n} |a_{ij}|}\right\}, \quad i \in N. \tag{1}$$

For a strictly diagonally dominant M-matrix, Cheng and Huang [4] presented the following result:

$$\|A^{-1}\|_\infty \leqslant \frac{1}{a_{11}(1-u_1d_1)} + \sum_{i=2}^{n}\left[\frac{1}{a_{ii}(1-u_id_i)}\prod_{j=1}^{i-1}\left(1 + \frac{u_j}{1-u_jd_j}\right)\right]. \tag{2}$$

A matrix $A = (a_{ij}) \in R^{n\times n}$ is a weakly chained diagonally dominant (*wcdd*) matrix if A is diagonally dominant, i.e., $|a_{ii}| \geqslant \sum_{j=1,j\neq i}^{n} |a_{ij}|$ for each $i \in N$ and where $J(A) = \left\{i \in N \| a_{ii}| > \sum_{j=1,j\neq i}^{n} |a_{ij}|\right\} \neq \phi$, and for each $i \notin J(A)$ there is a sequence of nonzero elements of A of the form $a_{i,i_1}, a_{i_1 i_2}, \ldots, a_{i_r j}$ with $j \in J(A)$.

We refer to [5] and [9] for more details of weakly chained diagonally dominant matrices.

Some practical problems such as those in digital circuit dynamics are related to *wcdd* matrices. Shivakumar et al. [5] first provided the following results in the infinity norm of the inverse of *wcdd* M-matrices

$$\|A^{-1}\|_\infty \leqslant \sum_{i=1}^{n}\left[a_{ii}\prod_{j=1}^{i}(1-u_j)\right]^{-1}, \tag{3}$$

where $u_j < 1,\ \forall j \in N$. Subsequently, Li [6] obtained the following bound for $\|A^{-1}\|_\infty$ of a *wcdd* M-matrix $A = (a_{ij})$ with $a_{kk}(1+l_k) > s_k(A),\ \forall k \in N$

$$\|A^{-1}\|_\infty \leqslant \sum_{i=1}^{n}\prod_{k=1}^{i}\frac{h_k}{a_{kk}(1+l_k) - s_k(A)}, \tag{4}$$

where $s_n(A) = R_n(A)$, $s_k(A) = \sum_{i=1}^{k-1} |a_{ki}| + \sum_{i=k+1}^{n} |a_{ki}| \frac{s_i(A)}{|a_{ii}|}$, $k = n-1, \ldots, 1$; $h_1 = 1$, $h_k = r_{k-1}(A) - s_{k-1}(A)$, $k = 2, \ldots, n$.

These above results can be applied to estimate the lower bound of the minimal eigenvalue of a *wcdd* M-matrix A and the condition number of a matrix.

In this paper, a new upper bound for $\|A^{-1}\|_\infty$ of *wcdd* M-matrices is further discussed, whose effectiveness will be shown by numerical examples.

2. Estimation for an upper bound of $\|A^{-1}\|_\infty$

Before a new upper bound is presented, some lemmas and results are given.

Lemma 1 [5]. *If $A = (a_{ij})$ is an $n \times n$ wcdd matrix and $A^{-1} = (\alpha_{ij})$, then for $i \neq j$*

$$|\alpha_{ij}| \leqslant \rho_i |\alpha_{jj}| \leqslant |\alpha_{jj}|,$$

and if $i \in J(A)$,

$$\frac{1}{|a_{ii}|(1+\rho_i)} \leqslant |\alpha_{ii}| \leqslant \frac{1}{|a_{ii}|(1-\rho_i)}.$$

The same result for a strictly diagonally dominant matrix was provided by Ostrowski [8].

Lemma 2. *If $A = (a_{ij})$ is an $n \times n$ wcdd M-matrix and $A^{-1} = (\alpha_{ij})$, then for $i \neq j$*

$$|\alpha_{ij}| \leqslant \frac{|a_{ij}| + \sum_{k \neq i,j} |a_{ik}| \rho_k}{|a_{ii}|} |\alpha_{jj}|, \quad i \neq j, \tag{5}$$

and when $j = 1$, we have

$$|\alpha_{i1}| \leqslant \frac{|a_{i1}| + \sum_{k \neq 1,i} |a_{ik}| \rho_k}{|a_{ii}|} |\alpha_{11}| \leqslant p_1 |\alpha_{11}| \leqslant |\alpha_{11}|, \quad i \neq 1, \tag{6}$$

and

$$\frac{1}{|a_{11}|(1+\rho_1 p_1)} \leqslant |\alpha_{11}| \leqslant \frac{1}{|a_{11}|(1-\rho_1 p_1)}. \tag{7}$$

Proof. Let

$$\rho_i(\varepsilon, 1) = \begin{cases} \frac{\sum_{k \neq i} |a_{ik}| + \varepsilon}{|a_{ii}|}, & i \in J(A), \\ 1, & i \in N, i \notin J(A), \end{cases}$$

where $\varepsilon > 0$ is sufficiently small such that $0 < \rho_i(\varepsilon, 1) \leqslant 1$ for $i \in N$. Let

$$D_j(\varepsilon, 1) = \mathrm{diag}(\rho_1(\varepsilon, 1), \ldots, \rho_{j-1}(\varepsilon, 1), 1, \rho_{j+1}(\varepsilon, 1), \ldots, \rho_n(\varepsilon, 1)), \quad j \in N.$$

Obviously, the matrix $AD_j(\varepsilon, 1)$ is a *wcdd* matrix when $D_j(\varepsilon, 1) = \mathrm{diag}(1, \ldots, 1) = I$, and the matrix $AD_j(\varepsilon, 1)$ is a strictly diagonally dominant matrix when $D_j(\varepsilon, 1) \neq I$. No matter what $D_j(\varepsilon, 1)$ is, the matrix $AD_j(\varepsilon, 1)$ is always a *wcdd* matrix. Therefore, by Lemma 1, for the matrix $AD_j(\varepsilon, 1)$, we have

$$\frac{|\alpha_{ij}|}{\rho_i(\varepsilon, 1)} \leqslant \frac{|a_{ij}| + \sum_{k \neq i,j} |a_{ik}| \rho_k(\varepsilon, 1)}{|a_{ii}| \rho_i(\varepsilon, 1)} |\alpha_{jj}|, \quad i \neq j,$$

i.e.,

$$|\alpha_{ij}| \leqslant \frac{|a_{ij}| + \sum_{k \neq i,j} |a_{ik}| \rho_k(\varepsilon, 1)}{|a_{ii}|} |\alpha_{jj}|, \quad i \neq j.$$

As $\varepsilon \to 0$, we obtain

$$|\alpha_{ij}| \leqslant \frac{|a_{ij}| + \sum_{k\neq i,j} |a_{ik}|\rho_k}{|a_{ii}|}|\alpha_{jj}|, \quad i \neq j,$$

and letting $j = 1$, we get

$$|\alpha_{i1}| \leqslant \frac{|a_{i1}| + \sum_{k\neq i,1} |a_{ik}|\rho_k}{|a_{ii}|}|\alpha_{11}|, \quad i \neq 1,$$

both of which prove (5) and the leftmost inequality of (6). Using the definitions of p_k and b_k, we have $\frac{|a_{i1}|+\sum_{k\neq i,1} |a_{ik}|\rho_k}{|a_{ii}|} \leqslant p_1 \leqslant 1$, from which one may deduce the inequality (6). From $A^{-1}A = I$ we have

$$\alpha_{11}a_{11} + \sum_{j=2}^{n} a_{1j}\alpha_{j1} = 1.$$

Hence

$$\begin{aligned}|\alpha_{11}a_{11}| &\leqslant 1 + \sum_{j=2}^{n} |a_{1j}\alpha_{j1}| \\ &\leqslant 1 + p_1|\alpha_{11}|\sum_{j=2}^{n} |a_{1j}|,\end{aligned}$$

which implies that

$$|\alpha_{11}|\left(|a_{11}| - p_1\sum_{j=2}^{n}|a_{1j}|\right) \leqslant 1,$$

resulting in

$$|\alpha_{11}| \leqslant \frac{1}{|a_{11}|(1-\rho_1 p_1)}.$$

The proof of the left inequality of (7) is analogous. □

The same results as (5) for strictly diagonally dominant matrices can be found in [7].

For nonempty index set $\beta^{(k)} \subseteq N$, we denote $A[\beta^{(k)}]$ as the submatrix of A whose rows and columns are indexed by $\beta^{(k)}$. By $A^{(k)}$ we denote $A^{(k)} = A[\alpha^{(k)}]$, where $\alpha^{(k)} = \{k+1, \ldots, n\}$; for instance, $A^{(1)}$ is the submatrix of A obtained by deleting the first row and the first column of A.

Recall that $A = (a_{ij}) \in R^{n\times n}$ is an L-matrix if for all $i, j \in N$ with $i \neq j$, $a_{ij} \leqslant 0$ and $a_{ii} > 0$.

Lemma 3 [5]. *A wcdd L-matrix is a nonsingular M-matrix.*

Lemma 4 [5]. *If $A = (a_{ij})$ is an $n \times n$wcdd M-matrix, then $B = A^{(1)}$ is an $(n-1)\times(n-1)$wcdd M-matrix. (i.e., $B^{-1} = (\beta_{ij})$ exists and $\beta_{ij} \geqslant 0,\ i, j = 2, 3, \ldots, n$).*

Let A be an $n \times n$ *wcdd* M-matrix, $B = A^{(1)}$, then by Lemmas 3 and 4, A and B are nonsingular, $B^{-1} = (\beta_{ij})$, $\beta_{ij} \geqslant 0$. Now we partition A and A^{-1} into the following block forms:

$$A = (a_{ij}) = \begin{bmatrix} a_{11} & x^T \\ y & B \end{bmatrix}, \quad A^{-1} = (\alpha_{ij}) = \begin{bmatrix} \alpha_{11} & u^T \\ v & C \end{bmatrix},$$

where $x^T = (a_{12}, \ldots, a_{1n})$, $y^T = (a_{21}, \ldots, a_{n1})$, $u^T = (\alpha_{12}, \ldots, \alpha_{1n})$ and $v^T = (\alpha_{21}, \ldots, \alpha_{n1})$. By expanding $AA^{-1} = I$, for $i, j = 2, \ldots, n$, it is easy to check that $\alpha_{11}\Delta = 1$, where

$$\Delta = a_{11} - x^T B^{-1} y = a_{11} - \sum_{k=2}^{n} a_{1k}\left(\sum_{i=2}^{n}\beta_{ki}a_{i1}\right).$$

Hence $\Delta \neq 0$ and $\alpha_{11} = \frac{1}{\Delta}$. Furthermore, $v = -\Delta^{-1}B^{-1}y$, which gives

$$\alpha_{i1} = \alpha_{11}\sum_{k=2}^{n}\beta_{ik}(-a_{k1}). \tag{8}$$

Similarly, from $C = B^{-1}(I - yu^T)$ and $u^T = -\Delta^{-1}x^TB^{-1}$, we get

$$\alpha_{1j} = \alpha_{11}\sum_{k=2}^{n}\beta_{kj}(-a_{1k}), \tag{9}$$

and

$$\alpha_{ij} = \beta_{ij} + \alpha_{1j}\sum_{k=2}^{n}\beta_{ik}(-a_{k1}).$$

Theorem 1. *Let A be an $n \times n$ wcdd M-matrix and $B = A^{(1)}, A^{-1} = (\alpha_{ij})_{n\times n}$ and $B^{-1} = (\beta_{ij})_{(n-1)\times(n-1)}$ and assume that $\rho_1 p_1 < 1$. Then*

$$\|A^{-1}\|_\infty \leqslant \max\left\{\frac{1}{a_{11}(1-\rho_1p_1)} + \frac{\rho_1\|B^{-1}\|_\infty}{1-\rho_1p_1}, \frac{p_1}{a_{11}(1-\rho_1p_1)} + \frac{\|B^{-1}\|_\infty}{1-\rho_1p_1}\right\}. \tag{10}$$

Proof. Let

$$r_i = \sum_{k=1}^{n}\alpha_{ik} = \alpha_{ii} + \sum_{k\neq i}\alpha_{ik}, \quad i = 1,\ldots,n,$$
$$M_1 = \|A^{-1}\|_\infty, \quad M_2 = \|B^{-1}\|_\infty.$$

Then $M_1 = \max\{r_i, 1 \leqslant i \leqslant n\}$ and $M_2 = \max\left\{\sum_{k=2}^{n}\beta_{ik}, 2 \leqslant i \leqslant n\right\}$. By Lemma 2 and (9),

$$\begin{aligned}
r_1 &= \alpha_{11} + \sum_{k=2}^{n}\alpha_{1k}\\
&= \alpha_{11} + \sum_{k=2}^{n}\alpha_{11}\sum_{p=2}^{n}\beta_{pk}(-a_{1p})\\
&= \alpha_{11} + \alpha_{11}\sum_{p=2}^{n}(-a_{1p})\sum_{k=2}^{n}\beta_{pk}\\
&\leqslant \alpha_{11} + \alpha_{11}\sum_{p=2}^{n}(-a_{1p})M_2\\
&\leqslant \frac{1}{a_{11}(1-\rho_1p_1)}(1 + a_{11}\rho_1M_2).
\end{aligned}$$

When $2 \leqslant i \leqslant n$, from (6) and (8), we get

$$\alpha_{i1} \leqslant p_1\alpha_{11},$$
$$\begin{aligned}
\alpha_{ij} &= \beta_{ij} + \alpha_{1j}\sum_{k=2}^{n}\beta_{ik}(-a_{k1})\\
&= \beta_{ij} + \frac{1}{\alpha_{11}}\alpha_{i1}\alpha_{1j}\\
&\leqslant \beta_{ij} + \alpha_{1j}p_1.
\end{aligned}$$

Thus, for $2 \leqslant i \leqslant n$, we obtain

$$\begin{aligned}
r_i &= \alpha_{i1} + \sum_{j=2}^{n} \alpha_{ij} \\
&\leqslant p_1\alpha_{11} + \sum_{j=2}^{n} (\beta_{ij} + \alpha_{1j}p_1) \\
&= p_1\alpha_{11} + \sum_{j=2}^{n} \beta_{ij} + \sum_{j=2}^{n} \alpha_{1j}p_1 \\
&\leqslant p_1 r_1 + M_2 \\
&\leqslant p_1 \frac{1 + a_{11}\rho_1 M_2}{a_{11}(1-\rho_1 p_1)} + M_2 \\
&= \frac{p_1}{a_{11}(1-\rho_1 p_1)} + \left(1 + \frac{\rho_1 p_1}{1-\rho_1 p_1}\right) M_2,
\end{aligned}$$

consequently,

$$\begin{aligned}
M_1 &= \max\{r_1, r_i|_{2 \leqslant i \leqslant n}\} \\
&\leqslant \max\left\{\frac{1 + a_{11}\rho_1 M_2}{a_{11}(1-\rho_1 p_1)}, p_1 \frac{1 + a_{11}\rho_1 M_2}{a_{11}(1-\rho_1 p_1)} + M_2\right\} \\
&= \max\left\{\frac{1}{a_{11}(1-\rho_1 p_1)} + \frac{\rho_1}{1-\rho_1 p_1} M_2, \frac{p_1}{a_{11}(1-\rho_1 p_1)} + \frac{1}{1-\rho_1 p_1} M_2\right\}.
\end{aligned} \tag{11}$$

According to (11), (10) follows. $\square$

Theorem 2. *Let A be an $n \times n$ wcdd M-matrix, and assume that $u_k p_k < 1$, $k = 1, \ldots, n$. Then*

$$\begin{aligned}
\|A^{-1}\|_\infty \leqslant \quad & \max\left\{\tfrac{1}{a_{11}(1-u_1p_1)} + \textstyle\sum_{i=2}^{n}\left[\tfrac{1}{a_{ii}(1-u_ip_i)} \prod_{j=1}^{i-1}(\tfrac{u_j}{1-u_jp_j})\right],\right. \\
& \left.\tfrac{p_1}{a_{11}(1-u_1p_1)} + \textstyle\sum_{i=2}^{n}\left[\tfrac{p_i}{a_{ii}(1-u_ip_i)} \prod_{j=1}^{i-1}\left(\tfrac{1}{1-u_jp_j}\right)\right]\right\}.
\end{aligned} \tag{12}$$

Proof. Apply induction with respect to k to $A^{(k)}$ with Theorem 1.

Note that $\rho_1 = u_1, u_n = 0$ and $p_n = 1$. $\square$

Remark 1. With the upper and lower bounds of the smallest eigenvalue $q(A)$ given in [5], our new lower bound of $q(A)$ is established with (10) and (12).

Remark 2. From (12), we can assume that

$$\|A^{-1}\|_\infty \leqslant \frac{1}{a_{11}(1-u_1p_1)} + \sum_{i=2}^{n}\left[\frac{1}{a_{ii}(1-u_ip_i)} \prod_{j=1}^{i-1}\left(\frac{u_j}{1-u_jp_j}\right)\right]. \tag{13}$$

When $n = 1, 2$, by actual computation, it is known that (13) is smaller than or equal to (4).

When $n \geqslant 3$, for $k = 1, \ldots, n-1$, we have

$$\begin{aligned}
\frac{u_k}{1-u_kp_k} &= \frac{\sum_{i=k+1}^{n}|a_{ki}|}{a_{kk} - \sum_{i=k+1}^{n}|a_{ki}|p_k}, \\
\frac{h_{k+1}}{a_{kk} + a_{kk}l_k - s_k(A)} &= \frac{a_{kk} + \sum_{i=k+1}^{n}|a_{ki}| - \sum_{i=k+1}^{n}|a_{ki}|\frac{s_i(A)}{a_{ii}}}{a_{kk} - \sum_{i=k+1}^{n}|a_{ki}|\frac{s_i(A)}{a_{ii}}}.
\end{aligned}$$

Obviously, $u_k < h_{k+1}$, and p_k is becoming decreasing while $\frac{s_i(A)}{a_{ii}}$ remains constant when k is increasing. So that $\frac{s_i(A)}{a_{ii}}$ is much larger than p_k. Therefore, we have

$$\frac{u_k}{1-u_kp_k} \leqslant \frac{h_{k+1}}{a_{kk}+a_{kk}l_k-s_k(A)},$$

and

$$\frac{1}{a_{kk}-\sum_{i=k+1}^{n}|a_{ki}|p_k} \leqslant \frac{1}{a_{kk}-\sum_{i=k+1}^{n}|a_{ki}|\frac{s_i(A)}{a_{ii}}},$$

when k is larger in practical cases. However, when k is smaller accordingly, it is possible that

$$\frac{u_k}{1-u_kp_k} \geqslant \frac{h_{k+1}}{a_{kk}+a_{kk}l_k-s_k(A)},$$

and

$$\frac{1}{a_{kk}-\sum_{i=k+1}^{n}|a_{ki}|p_k} \geqslant \frac{1}{a_{kk}-\sum_{i=k+1}^{n}|a_{ki}|\frac{s_i(A)}{a_{ii}}}.$$

If $\|A^{-1}\|_\infty \leqslant \frac{p_1}{a_{11}(1-u_1p_1)} + \sum_{i=2}^{n}\left[\frac{p_i}{a_{ii}(1-u_ip_i)}\prod_{j=1}^{i-1}\left(\frac{1}{1-u_jp_j}\right)\right]$, the discussion is analogous.

3. Examples

Example 1. Let

$$A=\begin{bmatrix}4 & -1 & -1\\ -2 & 5 & -3\\ -1 & -2 & 4\end{bmatrix},\quad \|A^{-1}\|_\infty = 1.11,\quad J(A)=\{1,3\}.$$

It is easy to verify that A is a weakly chained diagonally dominant M-matrix. We have

$$\|A^{-1}\|_\infty \leqslant 2.75 \quad (\text{by } (3)),$$
$$\|A^{-1}\|_\infty \leqslant 2.04 \quad (\text{by } (4)),$$
$$\|A^{-1}\|_\infty \leqslant 1.5 \quad (\text{by } (12)).$$

Example 2. Let

$$A=\begin{bmatrix}1 & 0 & -0.2\\ -0.2 & 1 & -0.1\\ -0.7 & -0.2 & 1\end{bmatrix},\quad \|A^{-1}\|_\infty = 2.33,\quad J(A)=\{1,2,3\}.$$

It is not difficult to verify that A is a strictly diagonally dominant M-matrix. We have

$$\|A^{-1}\|_\infty \leqslant 10 \quad (\text{by } (1)),$$
$$\|A^{-1}\|_\infty \leqslant 3.8878 \quad (\text{by } (2)),$$
$$\|A^{-1}\|_\infty \leqslant 4.03 \quad (\text{by } (3)),$$
$$\|A^{-1}\|_\infty \leqslant 3.97 \quad (\text{by } (4)),$$
$$\|A^{-1}\|_\infty \leqslant 2.5540 \quad (\text{by } (12)).$$

Acknowledgments

We would like to express our great thankfulness to the referees and editor Prof. Ludwig Elsner for your much constructive, detailed and helpful suggestions regarding revising this manuscript.

References

[1] A. Berman, R.J. Plemmons, Nonnegative Matrices in the Mathematics Sciences, SIAM Press, Philadelphia, 1994.
[2] R.A. Horn, C.R. Johnson, Topics in Matrix Analysis, Cambridge University Press, New York, 1985.
[3] R.S. Varga, On diagonal dominance arguments for bounding $\|A^{-1}\|_\infty$, Linear Algebra Appl. 17 (1976) 211–217.
[4] G.H. Cheng, T.Z. Huang, An upper bound for $\|A^{-1}\|_\infty$ of strictly diagonally dominant M-matrices, Linear Algebra Appl. 426 (2007) 667–673.
[5] P.N. Shivakumar, J.J. Williams, Q. Ye, Corneliu A. Marinov, On two-sided bounds related to weakly diagonally dominant M-matrices with application to digital dynamics, SIAM J. Matrix Anal. Appl. 17 (2) (1996) 298–312.
[6] W. Li, The infinity norm bound for the inverse of nonsingular diagonal dominant matrices, Appl. Math. Lett. 21 (2007) 258–263.
[7] H.B. Li, T.Z. Huang, S.Q. Shen, H. Li, Lower bounds for the minimum eigenvalue of Hadamard product of an M-matrix and its inverse, Linear Algebra Appl. 420 (2007) 235–247.
[8] A.M. Ostrowski, Note on bounds for determinants with dominant principal diagonal, Proc. Amer. Math. Soc. 3 (1952) 26–30.
[9] P.N. Shivakumer, K.H. Chew, Asufficient condition for nonvanishing of determinants, Proc. Amer. Math. Soc. 43 (1974) 63–66.